Instanton counting for 5d SYMs

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based on arXiv:1406.6793
with Joonho Kim, Seok Kim and Jaemo Park

Outline

- Motivation
- Z_{inst}
- Sp(1) theories w/o antisym
- Sp(1) theory for 6d SCFT
- Summary

Motivation

$\mathcal{N}$=1 Sp(N) gauge theories

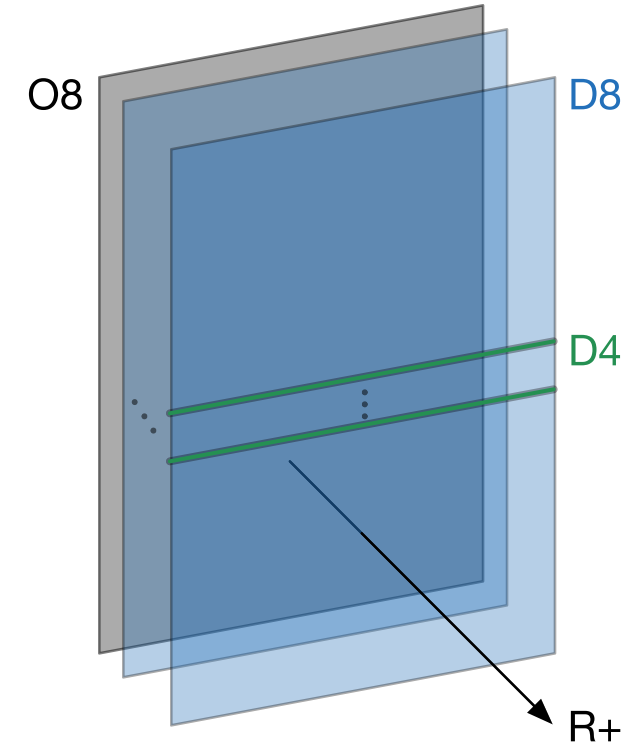
5d N=1 Sp(N) gauge theories

5-dimensional $\mathcal{N}$=1 Sp(N) gauge theory
with one antisym and Nf fund hypermultiplets

- nonrenormalizable -> effective field theory

Seiberg 96

- Nf=0,…,7: 5d SCFT at UV fixed point -> exhibits enhanced E_{Nf+1} global symmetry
- Nf=8: the circle compactification of 6d SCFT -> E_8 global symmetry
- engineered by type IIA string theory on $R^{8,1}\times R^+$



Instanton partition function

Instanton partition function

Instantons

- self-dual $$F_{mn} = \star_4 F_{mn} = \frac{1}{2}\epsilon_{mnpq}F_{pq}$$
- instanton charge $$k = \frac{1}{8\pi^2}\int_{\mathbb{R}^4} \mathrm{tr}(F\wedge F) \in \mathbb{Z}_+$$
- preserve half SUSY
- form marginal bound states with W-bosons

Instanton moduli space

- described by a nonlinear sigma model
- Small instanton singularities -> inherit the nonrenormalizability of the 5d theory
- UV completion -> ADHM gauged quantum mechanics

Atiyah, Hitchin, Drinfeld, Manin 78
Nekrasov 04

Instanton partition function -> ADHM QM index

Instanton partition function

The ADHM QM index

$$Z^k_{\mathrm{QM}}(\epsilon_1, \epsilon_2, \alpha_i, z) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i \Pi_i} e^{-zF}\right]$$

Matrix integral:

$$Z = \frac{1}{|W|}\oint Z_{\text{1-loop}} = \frac{1}{|W|}\oint Z_V \prod_\Phi Z_\Phi \prod_\Psi Z_\Psi$$

Questions?

- Which integration contour?
- $Z_{\mathrm{inst}}=Z_{\mathrm{QM}}$?

Instanton partition function

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Questions?

- Which integration contour? cf. avoiding contour issue by discarding antisym hyper for Sp(1)
  H. -C. Kim, S. Kim, K. Lee 12

- $Z_{\text{inst}}=Z_{\text{QM}}$?

Instanton partition function

The ADHM QM index

$$Z^k_{\rm QM}(\epsilon_1,\epsilon_2,\alpha_i,z) = {\rm Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i\Pi_i} e^{-zF}\right]$$

Matrix integral:

$$Z = \frac{1}{|W|}\oint Z_{\text{1-loop}} = \frac{1}{|W|}\oint Z_V \prod_\Phi Z_\Phi \prod_\Psi Z_\Psi$$

Questions?

- Which integration contour?
  -> Jeffrey-Kirwan residue. Benini, Eager, Hori, Tachikawa 13
- $Z_{\rm inst}$=$Z_{\rm QM}$?

Instanton partition function

The ADHM QM index

$$Z^k_{\rm QM}(\epsilon_1,\epsilon_2,\alpha_i,z) = {\rm Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i\Pi_i} e^{-zF}\right]$$

Matrix integral:

$$Z = \frac{1}{|W|}\oint Z_{\text{1-loop}} = \frac{1}{|W|}\oint Z_V \prod_\Phi Z_\Phi \prod_\Psi Z_\Psi = Z_{\rm inst}?$$

Questions?

- Which integration contour?
  -> Jeffrey-Kirwan residue.

- $Z_{\rm inst} = Z_{\rm QM}$?

Instanton partition function

The ADHM QM index

$$Z^k_{\mathrm{QM}}(\epsilon_1, \epsilon_2, \alpha_i, z) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i \Pi_i} e^{-zF}\right]$$

Matrix integral:

$$Z = \frac{1}{|W|}\oint Z_{1\text{-loop}} = \frac{1}{|W|}\oint Z_V \prod_\Phi Z_\Phi \prod_\Psi Z_\Psi$$

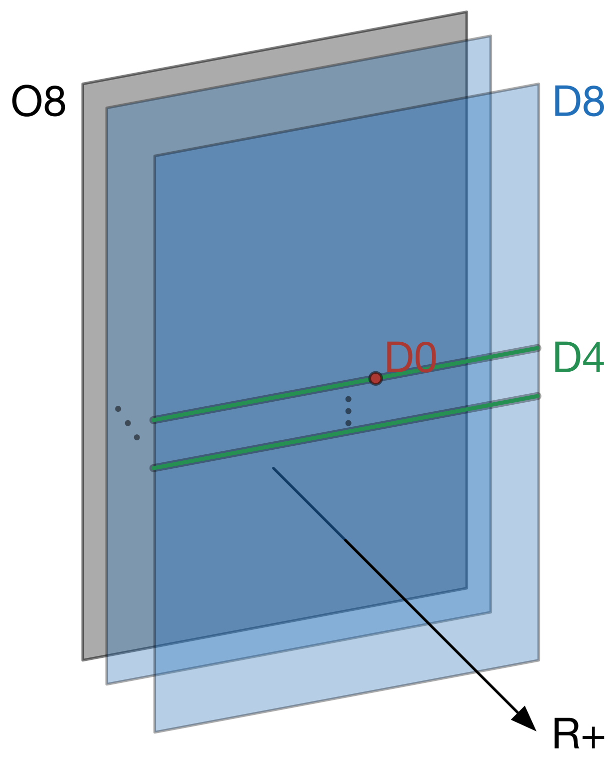
Questions?

- Which integration contour?
  -> Jeffrey-Kirwan residue.

- $Z_{\mathrm{inst}}=Z_{\mathrm{QM}}$?
  -> Not always. UV completion might give rise to extra degrees of freedom.

Z_{inst}

ADHM quantum mechanics



Douglas 96; Aharony, Hanany, Kol 97

$$\text{D0-D0} : O(k) \text{ antisymmetric} \quad (A_t, \varphi) \ , \quad (\bar{\lambda}^A_{\dot{\alpha}}, \underline{\lambda^a_\alpha})$$

$$O(k) \text{ symmetric} \quad (a_{\alpha\dot{\beta}}, \underline{\varphi_{aA}}) \ , \quad (\lambda^A_\alpha, \underline{\bar{\lambda}^a_{\dot{\alpha}}})$$

$$\text{D0-D4} : Sp(N) \times O(k) \text{ bif.} \quad (q_{\dot{\alpha}}) \ , \quad (\psi^A, \underline{\psi^a})$$

$$\text{D0-D8} : SO(2N_f) \times O(k) \text{ bif.} \quad (\underline{\Psi_l})$$

The ADHM QM index:

$$Z_{\text{QM}} = \frac{1}{|W|} \sum_{\phi_*} \text{JK-Res}(\mathbf{Q}(\phi_*), \eta) \ Z_{\text{1-loop}}(\phi, \epsilon_+, z)$$

$Z_{\text{inst}} = Z_{\text{QM}}$?

- noncompact Coulomb branch: lifted for Nf≤7
- part of Higgs branch: non 5-dimensional degrees of freedom

Extra string theory states

D0-D8-O8 bound states (unbounded from D4)

- captured by D0 quantum mechanics on D8-O8
- Dropping bifundamentals from D0-D4,

$$Z_{\rm QM} = \frac{1}{|W|}\sum_{\phi_*} \text{JK-Res}(\mathbf{Q}(\phi_*),\eta)\ Z_{\text{1-loop}}(\phi,\epsilon_+,z)$$

$$\begin{aligned}
Z_{N_f=0} &= \text{PE}\left[-\frac{t^2 q}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\right] \\
Z_{1\le N_f\le 5} &= \text{PE}\left[-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)} q\chi(y_i)^{SO(2N_f)}_{\mathbf{2^{N_f-1}}}\right] \\
Z_{N_f=6} &= \text{PE}\left[-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left(q\chi(y_i)^{SO(12)}_{\mathbf{32}} + q^2\right)\right] \\
Z_{N_f=7} &= \text{PE}\left[-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left(q\chi(y_i)^{SO(14)}_{\mathbf{64}} + q^2\chi(y_i)^{SO(14)}_{\mathbf{14}}\right)\right]
\end{aligned}$$

-> exactly the 0th order term of $Z_{\rm QM}$ expanded in the electric charge fugacity

Extra string theory states

D8-O8 system

- 9d SO(2Nf) SYM theory
- enhanced to E_{N_f+1} refering to string duality
- The perturbative index:

$$2\sinh\frac{\epsilon_1}{2}\cdot 2\sinh\frac{\epsilon_2}{2}\cdot 2\sinh\frac{m+\epsilon_+}{2}\cdot 2\sinh\frac{m-\epsilon_+}{2}\times\frac{1}{\left(2\sinh\frac{\epsilon_1}{2}\cdot 2\sinh\frac{\epsilon_2}{2}\cdot 2\sinh\frac{m+\epsilon_+}{2}\cdot 2\sinh\frac{m-\epsilon_+}{2}\right)^2}\times\chi^{SO(2N_f)}_{\mathbf{adj}}(y_i)^+$$

broken

Q^a_α , Q^A_α , $\bar{Q}^a_{\dot\alpha}$, $\bar{Q}^A_{\dot\alpha}$

the translations on R^8

the electric charges

$$f_{\text{pert}}=\frac{\chi^{SO(2N_f)}_{\mathbf{adj}}(y_i)^+}{2\sinh\frac{\epsilon_1}{2}\cdot 2\sinh\frac{\epsilon_2}{2}\cdot 2\sinh\frac{m+\epsilon_+}{2}\cdot 2\sinh\frac{m-\epsilon_+}{2}}=-\frac{t^2\chi^{SO(2N_f)}_{\mathbf{adj}}(y_i)^+}{(1-tu)(1-t/u)(1-tv)(1-t/v)}$$

Extra string theory states

The perturbative index for D8-O8

$$f_{\text{9d SYM}} = -\frac{t^2 \chi^{SO(2N_f)}_{\mathbf{adj}}(y_i)^+}{(1-tu)(1-t/u)(1-tv)(1-t/v)}$$

The nonperturbative index for D0-D8-O8

$$f_0 = -\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\ q$$

$$f_{N_f} = -\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\ q\chi(y_i)^{SO(2N_f)}_{\mathbf{2^{N_f-1}}}$$

$$f_6 = -\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[q\chi(y_i)^{SO(12)}_{\mathbf{32}} + q^2\right]$$

$$f_7 = -\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[q\chi(y_i)^{SO(14)}_{\mathbf{64}} + q^2\chi(y_i)^{SO(14)}_{\mathbf{14}}\right]$$

$$E_4 = SU(5) : \mathbf{24} \to \mathbf{1}_0 + \mathbf{15}_0 + \mathbf{4}_1 + \overline{\mathbf{4}}_{-1}$$

$$E_5 = SO(10) : \mathbf{45} \to \mathbf{1}_0 + \mathbf{28}_0 + (\mathbf{8_s})_1 + (\mathbf{8_s})_{-1}$$

$$E_6 : \mathbf{78} \to \mathbf{1}_0 + \mathbf{45}_0 + \mathbf{16}_1 + \overline{\mathbf{16}}_{-1}$$

$$E_7 : \mathbf{133} \to \mathbf{1}_0 + \mathbf{66}_0 + \mathbf{32}_1 + \mathbf{32}_{-1} + \mathbf{1}_2 + \mathbf{1}_{-2}$$

$$E_8 : \mathbf{248} \to \mathbf{1}_0 + \mathbf{91}_0 + \mathbf{64}_1 + \overline{\mathbf{64}}_{-1} + \mathbf{14}_2 + \mathbf{14}_{-2}$$

- E_{N_f+1} enhancement
  -> supports duality between I' & heterotic
- UV completion additionally captures 9d spectrum.

$$Z_{\text{inst}} = Z_{\text{QM}}/Z_{\text{string}}$$

Sp(1) theories w/o antisym

Sp(1) theories w/o antisym: revisited

Two classes of 5d rank N SCFTs
-> Sp(N) SYMs w/ or w/o an antisym hyper

Seiberg 96
Intriligator, Morrison, Seiberg 97

- 1st class: UV fixed points for $N_f \leq 7$, engineered by D4-D8-O8 or M-theory on CY3
- 2nd class: UV fixed points for $N_f \leq 2N+4$, engineered by M-theory on CY3
- For Sp(1), the two classes are expected to yield the same SCFTs.

The same SCFT but different string theory engineerings
-> The same Z_{inst} but different Z_{QM}'s

Sp(1) theories w/o antisym: revisited

Instantons in the CY engineering

- M2-branes wrapping 2-cycles
- signal of noncompact modulus: $Z_{\text{1-loop}}$ approaches a const for Nf=6.

-> for Nf=6, M2 can escape to infinity

$$Z_{\text{string}} = \mathrm{PE}\left[-\frac{(1+t^2)q^2}{2(1-tu)(1-t/u)}\right]$$

-> for Nf<6, no extra UV degrees of freedom

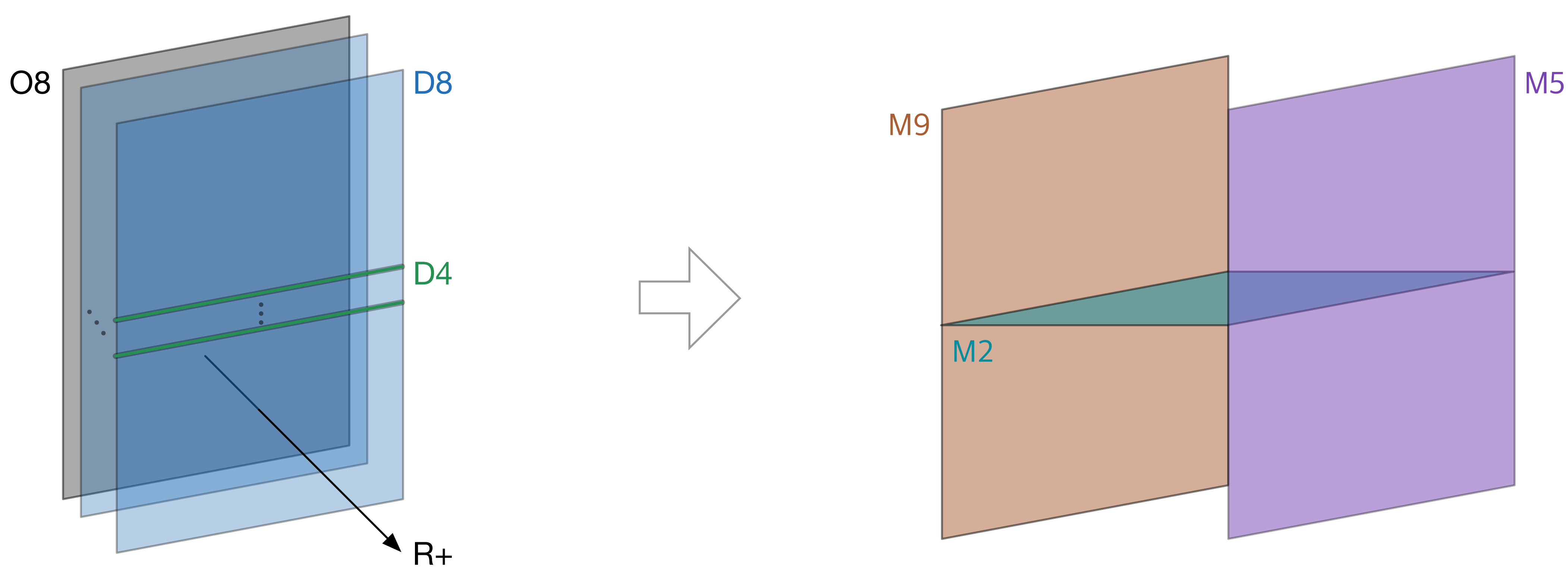
$$Z_{\text{string}} = \frac{Z_{\text{QM}}^{\text{w/o}}}{Z_{\text{inst}}^{\text{w/}}} = 1$$

Sp(1) theory for 6d SCFT

Sp(1) theory for 6d SCFT

Sp(1) theory with Nf = 8 (& antisym)

- 8 D8 + O8 -> zero D8-brane charge
- Uplift to M-theory on $R^{8,1} \times R^{+} \times S^{1}$ -> M9-M5 system
- The circle compactification of 6d (1,0) SCFT
- E-string Klemm, Mayr, Vafa 96



Sp(1) theory for 6d SCFT

Extra string states

- zero electric charge sector:

$$f = \left[\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-t/u)} - \frac{(t+t^3)(u+u^{-1}+v+v^{-1})}{2(1-tu)(1-t/u)(1-tv)(1-t/v)}\right]\frac{q^2}{1-q^2}$$
$$-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[\chi(y_i)^{SO(16)}_{\mathbf{120}}\frac{q^2}{1-q^2} + \chi(y_i)^{SO(16)}_{\mathbf{128}}\frac{q}{1-q^2}\right]$$

-> Not manifest E8 due to nonzero Wilson lines; y_8 -> $y_8 q$

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$$-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[\chi(y_i)^{SO(16)}_{\mathbf{120}}\frac{q^2}{1-q^2} + \chi(y_i)^{SO(16)}_{\mathbf{128}}\frac{q}{1-q^2}\right]$$

-> Not manifest E8 due to nonzero Wilson lines; y_8 -> $y_8 q$

- the second line + $f_{\rm pert} = -\frac{t^2\chi^{SO(2N_f)}_{\mathbf{adj}}(y_i)^+}{(1-tu)(1-t/u)(1-tv)(1-t/v)} = -\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[\chi^+_{\mathbf{91}} + y_8^2\chi^{SO(14)}_{\mathbf{14}}\right]$

⇨ $$-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[\chi^{E_8}_{\mathbf{248}}(y_i)\frac{q^2}{1-q^2} + \chi^{E_8}_{\mathbf{248}}(y_i)^+\right]$$

-> the KK tower of 10d E_8 SYM

Sp(1) theory for 6d SCFT

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- Type I SUGRA: dilaton ϕ, RR 2-form C_2, graviton $g_{\mu\nu}$, dilatino λ, gravitino ψ_μ

$$(\mathbf{1}\oplus\mathbf{28}\oplus\mathbf{35}_v)_{\text{boson}}\oplus(\mathbf{8}_s\oplus\mathbf{56}_s)_{\text{fermion}} = (\mathbf{8}_v\otimes\mathbf{8}_v)_{\text{sym}}\oplus(\mathbf{8}_v\otimes\mathbf{8}_c)\oplus(\mathbf{8}_c\otimes\mathbf{8}_c)_{\text{anti}}$$

$$\chi(\mathbf{8}_v) = (t+t^{-1})(u+u^{-1}+v+v^{-1})$$
$$\chi(\mathbf{8}_c) = -t^2-2-t^{-2}-(u+u^{-1})(v+v^{-1})$$
$$\chi(\mathbf{8}_s) = -(t+t^{-1})(u+u^{-1}+v+v^{-1})$$

+

the translations on R^8

$$\Rightarrow \quad -\frac{(t+t^3)(u+u^{-1}+v+v^{-1})}{(1-tu)(1-t/u)(1-tv)(1-t/v)}$$

Sp(1) theory for 6d SCFT

Extra string states

- zero electric charge sector:

$$f = \left[\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-t/u)} - \frac{(t+t^3)(u+u^{-1}+v+v^{-1})}{2(1-tu)(1-t/u)(1-tv)(1-t/v)}\right]\frac{q^2}{1-q^2}$$
$$- \frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left[\chi(y_i)^{SO(16)}_{\mathbf{120}}\frac{q^2}{1-q^2} + \chi(y_i)^{SO(16)}_{\mathbf{128}}\frac{q}{1-q^2}\right]$$

6d (1,0) SCFT index

$$Z_{\rm SCFT} = \frac{Z_{\rm QM}}{Z_{\rm string}}$$

- compared with E-string index Klemm, Mayr, Vafa 96; Haghighat, Lockhart, Vafa 14; J. Kim, S. Kim, K. Lee, J. Park, Vafa 14; Cai, Huang, Sun 14

Summary

- Instanton partition functions of 5d SYMs -> good observables for 5d/6d SCFTs
- Nonrenormalizability of 5d gauge theories -> singularities in instanton moduli space
- UV completion -> ADHM gauged quantum mechanics
- Matrix integral for ADHM index -> JK-residue, generally applicable to 1d N=2 theories
- ADHM index may capture extra UV degrees of freedom
- Different UV completions -> different Z_{QM}'s but the same Z_{inst}
- *String theory can be a guideline for handling nonrenoramlizable effective field theories*